Cross-sectoral interactions in Islamic equity markets☆

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ABSTRACT

Although it is essential for investors who want to comply with their religious obligations, cross-sectoral interaction in Islamic equity markets is an untouched subject in finance literature. Accordingly, this paper aims to investigate the interactions between the ten major sectors of Islamic equity markets by implementing the novel methodologies of dynamic conditional correlation (DCC) and dynamic equicorrelation (DECO) on Dow Jones Islamic Market sector indexes. We show that prior to the financialization period, firm fundamentals and real economic factors had an important role in driving the Islamic equity prices, however this role seemed to weaken in the last decade with the global financialization, leading to highly integrated Islamic equity sectors just as in the case of the conventional financial sectors. Moreover, this effect is emphasized further through financial contagion channels in the recent global financial crisis. Our findings thus suggest that Islamic equity indexes are also prone to global shocks hitting the world financial system, and investors should be cautious in interpreting and forecasting the interaction structure between Islamic equity sectors. Furthermore, our results do not support the decoupling hypothesis of the Islamic equity markets from the conventional financial system.

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1. Introduction

Given the complex and dynamic structure of financial markets, one of the challenges that investors face is the active or passive investing decision. Even though the answer is not certain, the amount of passively managed assets has been gaining momentum for the couple of decades. The popularity of passively managed assets, in turn, promotes the demand on index trading. In fact, Bhattacharya and Galpin (2011)
showed that the popularity of index portfolio has been increasing globally. Index investors, in general, favor strategic portfolio allocation and are inclined to trade in and out of the markets at the same time. This commonality in market timing results in increased correlation between indexes and consequently decreased diversification benefits. Markets become more synchronized and vulnerable hence the systemic risk level rises. The experience of the 2008 financial crisis showed that, index investors (who link various sectors) sharpen the increase in co-movements between sectors. Within this crises period, in an environment where even the benefits of portfolio diversification have been questioned, market players and academicians started to search for remedies and renovation solutions for the conventional financial system. In this process, Islamic finance emerged as a viable alternative due to its seemingly resilient structure and promising return performance.

In addition to the growing interest from the Western world especially after the 2008 financial crisis, the pace of demand in Islamic finance is growing as a result of the accumulation of pool of oil wealth, strong participation from Muslim investors combined with the keen willingness of regulators to support the development of Islamic markets. Total Islamic finance assets grew to an estimated USD 1.8 trillion by the end of 2013. The industry is estimated to chart a compound annual growth rate (CAGR) of 17.04% between 2009 and 2013. Similarly, the global Islamic funds industry has progressed tremendously; assets under management have grown from USD 29.2 billion in 2004 to more than USD 70 billion as of December 2013. From 2007 to 2012, the total assets managed by Islamic funds increased by a CAGR of 7.3%, while the mature conventional fund industry grew by 1.8% during the same period. According to data compiled by Thomson Reuters Zawya Islamic, USD 85.8 billion of sukuk was sold during the first eight months of 2014. More importantly, four countries (UK, Luxembourg, Hong Kong and South Africa) which do not have a predominantly Muslim population, issued their debut sukuk. These issuances manifest that not only Muslim countries utilized Islamic finance to raise funds. Furthermore, existing markets such as Malaysia, Saudi Arabia, Turkey and the UAE continue their efforts to facilitate growth of Islamic capital markets and flows of funds using Islamic finance instruments.

As the Islamic Law (Shari’ah) compliant products attract more investment, its reflection can be seen from the number of Islamic indexes created. Relatively easy access to Islamic indexes broke the domestic-centric structure of Islamic finance and helped its internationalization. Besides, Islamic fund managers highly benefited from the standardization of Shari’ah screening methodologies of Islamic equity indexes such that the indexes help to reduce research costs, mitigate compliance concerns of Muslim investors and expand the number of securities available to invest. Since the screening methodology has been standardized and sub-indexes have been created, investors have the opportunity to allocate their portfolio in distinctive sectors. Moreover, by investing in passively managed Islamic index funds, Muslim investors do not necessarily need to sacrifice returns to comply with their religious obligations.

Recognition of potential diversification benefits from Islamic finance products and standardization of Shari’ah compliance process for the Islamic indexes stimulated the growth of Islamic finance index investments which, in turn, gave rise to financialization among stocks which are covered by these indexes. Globalization, advances in information dissemination, technological improvements, integration of emerging markets and easy flow of capital between markets precipitated the financialization process. By financialization, we mean the expanded role of financial market developments, risk appetite for financial assets, investment strategies and

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1 “Renovation solutions for the conventional financial system” refers to restructuring of the financial system. As renovation means making changes and repairs so that it is back in “good” condition (not necessarily the same old condition), it is used to refer to the efforts made in financial markets during/after a financial crisis period.


3 Sukuk is the Arabic name for financial certificates, but commonly refers to the Islamic equivalent of bonds.

4 For details, see http://www.zawya.com/story/New_sovereign_issues_usher_in_new_era_for_global_sukuk_industry-ZAWYA20140911074237/?zawyaemailmarketing.

5 To ensure compliance with the Shari’ah rules, Islamic indexes are governed by Shari’ah advisory boards whose role is mainly to review the underlying asset or structure and issue opinions as to their compliance. Stocks are selected by filtering the stock universe through rules regarding business activities and financial ratios. The important point is that the stock universe is not limited by the companies located in Muslim countries. Instead, without considering religious beliefs, indexes include shares from both Muslim or non-Muslim countries provided that they are Sharia-compliant. In fact, as of September 2014, country allocation of Dow Jones Islamic Index to US was 59.08%. 
the investors’ behavior on valuing stocks, thus increased correlation among them. In this context, we attempt to find out whether Islamic finance sector indexes have been affected by this financialization trend and have become more exposed to macroeconomic and financial shocks.

In line with the intensifying attention, awareness and demand for Islamic finance products and markets, the number of publications relating Islamic finance has also been mounting up. To be one of the early corners of this movement, we have investigated the correlations between Islamic sector indexes. Considering the importance of diversification in portfolio management, the results have important implications for faith-based investors and portfolio managers: due to several reasons, Islamic equities are not strictly immune to widespread impacts of interest rate changes or conventional financial system’s conditions. This situation brings out the inevitable necessity for portfolio diversification for the investors who want to reconcile faith with finance. Regarding equity investment strategies in this context, one of the first things that comes to mind is to construct a portfolio that is well diversified among different sectors. Such an aggregate portfolio may be the key to become immune to common risks of conventional finance. Therefore, outcomes of this study are especially crucial for the ones who are just building up their portfolios from the Islamic sector indexes. This may be the case for investors who are passively investing only or overwhelmingly in Islamic equities. Finally, the significance of diversification increases during crises hence the data set used in this study covers a period from January 3, 1996 to July 9, 2014 which is long enough to examine the effects of dot-com bubble, Lehman’s collapse and the following global financial crisis, and Eurozone sovereign debt crisis.

In this study, we analyze ten major Dow Jones Islamic equity sector indexes and accordingly 45 correlation pairs. Regarding modeling correlations, the corner stone study in this area is Bollerslev (1990)’s work in which he proposed a model which had conditional variances and covariances but the correlation matrix was time invariant. Engle (2002), by generalizing the work of Bollerslev (1990), overcame this drawback and embedded time-varying behavior of correlation (dynamic conditional correlation—DCC) into the model. In his study, bivariate models showed sensible empirical results, and DCC models and its variations widely employed in empirical analysis thereafter. Consequently, here in this study to model dynamic correlations between Islamic equity sector pairs, we used the novel model of Aielli (2013)’s consistent dynamic conditional correlation (cDCC).

Moreover, we also investigate the cross-sectoral correlation using dynamic equicorrelation (DECO) model of Engle and Kelly (2012). DECO represents the correlation degree of a group of assets with single dynamic correlation coefficient. Instead of analyzing and comparing each and every pairwise correlations to understand the level of financialization, by employing DECO we have the opportunity to observe the big picture of relations among Islamic equity sector indexes for the whole period. On top of that, for a more clear picture, we use the filtering procedure of Hodrick and Prescott (1997) to eliminate the noise in the dynamic equicorrelation and focus on the trend correlation component. Furthermore, the breaks/shifts in the dynamic equicorrelations are endogenously detected by the novel methodology of penalized contrast functions Lavielle (2005). Accordingly, we can clearly identify and comment on the events that create abrupt changes in the correlation levels.

To the best of our knowledge, our study is the first in literature that examines correlations of Islamic equity sector indexes. Besides, the use of cDCC and DECO methods have valuable inferences for both investors and portfolio managers since they can both look at each pair individually or interpret the convergence or divergence of all sectors. Although it is out of scope of this study, on top of all, the cross-sectoral relationship of conventional equity sector indexes is also examined and compared with its Islamic counterpart. This enabled us to show how the diversification benefits from creating a portfolio only from sector indexes have evolved for both Islamic and conventional indexes.

Our results show that prior to the financialization period, firm fundamentals and real supply/demand factors had an important role in driving the related Islamic equity prices, however the impact of the fundamentals on prices seemed to disappear in the last decade with the global financialization, leading to a high level of sensitivity in Islamic equity prices to the information captured in other asset prices and, inevitably, highly integrated Islamic equity sectors just as in the case of the conventional financial system. Moreover, this effect is emphasized further through financial contagion channels in the recent global financial crisis.

The remainder of this paper is organized as follows. Section 2 presents the existing literature. Section 3 describes the data set and explains the methodology that we followed. Section 4 displays and discusses the empirical results. Finally, Section 5 contains a summary and some concluding remarks.
2. Literature review

The academic literature on co-movement among international equity markets is voluminous, however the correlations of sectors and their portfolio diversification implications have not been explored adequately. Significance of impact of industry factors on the stock prices is first explored by King (1966). Following his study, Lessard (1974) found only a minor contribution from sector effects to equity returns. Schwartz and Altman (1973) and Livingston (1977) studied the influence of industries on volatility of equity shares and attained significant results. Roll (1992) suggested that each country’s sectoral structure plays a major role in explaining stock price behavior.

For quite a long time, it was believed that cross country diversification benefits outweigh sectoral diversification. On the other hand, studies of Baca et al. (2000), Cavaglia et al. (2000), and Moerman (2008) claimed that the gains from sectoral diversification is higher compared to diversification over countries. While Berben and Jansen (2005) analyzed sectors’ time varying correlations between international equity markets, Fasnacht and Louberge (2007) compared the sector correlations within and between international markets. Authors covered sector returns from January 1973 to March 2006 for seven major stock markets (the US, the UK, France, Germany, Switzerland, Canada and Japan) and employed DCC-GARCH methodology in order to capture differences in the behavior of sector correlation coefficients. They showed that the market correlations are on the average higher than correlations at the sectoral level and they attributed this case to the presence of the sector correlations within each country. Moreover, they suggested that the stock market correlations have increased for the analyzed period and claimed that this increase was largely driven by an increase in the sector correlations between countries.

Meric et al. (2008) investigated the linkages between ten sectors in the US, UK, Germany, France and Japan by discriminating between bull market (September 1997–March 2000) and bear market (March 2000–October 2002). Their results revealed that the best portfolio diversification opportunities are in utilities sector in both periods. Moreover, while in bear market the least diversification benefit are from information technology, finance, cyclical consumer goods and non-cyclical services; in bull market, information technology and financial sectors provide the least diversification opportunity.

Some of the studies focused more on the country or the region specific sector correlations. Arbelaez et al. (2001) investigated the Columbian market between 1988 and 1994 and showed that sector indexes are correlated and both the short-term and long-term linkages have become stronger over time. Wang et al. (2005) examined the Chinese markets and found relatively limited diversification benefits from sector-level investment. Similarly, Cao et al. (2013) analyzed the Chinese stock markets for the period between July 2007 and December 2012 during which the equity market experienced drastic shock periods (2007–2008) and general ups and downs (2009–2012). According to their results, correlations between sectors with the strong up and down periods showed higher correlation compared to the second period. Besides, similarity analysis showed that financials, industrials and energy have a high correlation with the whole market.

Balli and Balli (2011) modeled Euro-wide sectoral equity indexes between 1992 and 2007, and found that financial sector indexes are being increasingly affected by the aggregate Euro equity index fluctuations observed after the start of the Economic and Monetary Union. They also suggest that diversification across Euro sectoral equity indexes is more preferable than diversification of the portfolio across Euro nation indexes after the start of the Euro. Balli et al. (2013) examined the GCC equity markets using mean-variance portfolio approach and found that portfolios diversified with a mix of sector indexes produce better results than portfolios consisting of pure GCC national equity indexes. In contrast, Balli et al. (2014) analyzed ASEAN sectoral and national equity indexes using similar methodologies and came up with the result of relatively better performance of national diversification against sectoral diversification.

Regarding the reflection of these studies on Islamic finance literature, to the best of our knowledge, there is no study on the correlation of Islamic sector indexes. Although, as the investments in Islamic finance have accelerated and attention from academic world has also increased, most of the research accomplished until now is about the comparison of performance of Islamic finance instruments with conventional ones (Hussein and Omran, 2005; Girard and Hassan, 2008; Jawadi et al., 2014; Al-Khazali et al., 2014; Ho et al., 2014; Ashraf and Mohammad, 2014; Kamil et al., 2014).

Chong and Liu (2009) claimed that Islamic deposits are not interest-free and found that Islamic banking is not very different from conventional banking from the perspective of profit loss sharing. As stated by
Shamsuddin (2014) and Chong and Liu (2009), Islamic indexes are influenced by interest rate changes since those indexes are not strictly Shari’ah compliant due to permissible degree of financial leverage in the balance sheet of companies. Shamsuddin (2014)’s results indicate that while the aggregate portfolio of DJ Islamic stocks is immune to the change in the interest rate and volatility of interest rate, DJ Islamic sector portfolios show evidence of sensitivity to interest rate level. He suggests investing in a well-diversified Islamic equity portfolio similar to DJ Islamic market index in order to immunize portfolios against interest rate risk. He also warned the investors who want to implement sector rotation strategy against interest rate correlation of sectors.

The literature shows that there are considerable amount of studies on cross-sectoral interactions in conventional equity markets and their implications on investment decisions, however, such kind of studies for its Islamic counterpart is virtually none.

3. Data and methodology

The data used in our study covers a period from January 3, 1996 to July 9, 2014 and is obtained from Bloomberg history server. We use the weekly (Wednesdays') closing prices of ten major Islamic equity sector indexes (quoted in US dollar) disseminated by the Dow Jones data-feed of the Chicago Board of Trade (see Fig. 1).6

Mainly, these indexes are created for the people who wish to invest in worldwide companies/sectors that follow Islamic investment guidelines. As of September 2014, Dow Jones Islamic Market indexes include 2578 companies from 58 countries representing 10 main economic sectors. The Dow Jones Islamic indexes exclude companies whose primary businesses are alcohol, pork-related products, conventional financial services, tobacco, entertainment (e.g., gambling, hotels, and pornography), weapons and defense. After the first screening process, stocks are required to pass the second screening process regarding financial ratios. To be eligible for the Dow Jones Islamic indexes, the following financial ratios must be lower than 33%: (i) total debt divided by trailing 24-month average market capitalization, (ii) the sum of a company’s cash and interest-bearing securities divided by trailing 24-month average market capitalization, and (iii) accounts receivables divided by trailing 24-month average market capitalization. The Dow Jones Islamic indexes are monitored by a supervisory board of Islamic scholars who review the composition of these indexes quarterly and decide the composition change of these indexes (i.e., exclusion or inclusion of stocks) that are implemented on the third Friday in March, June, September, and December. With the purpose of constructing investable indexes, stocks that are readily accessible and well traded are chosen.7

The complete sector list and the corresponding tickers are given in Table 1.

For each sector $i$, we use the log-returns i.e. $R_{it} = \ln(P_{it}/P_{i,t-1})$ for the weekly changes where $P_{it}$ is the closing price of the sector $i$ on week $t$ (see Fig. 2). Initial analysis reveal that there are no serious effects of serial correlation or lingering random shocks, therefore we first estimate the following mean equation to obtain the zero mean residuals:

$$r_t = \mu + \varepsilon_t$$ (1)

where $r_t = [r_{1,t}, \ldots, r_{n,t}]'$ is the vector of $n$ sector returns, $\mu$ is a vector of constants with length $n$ and $\varepsilon_t = [\varepsilon_{t,1}, \ldots, \varepsilon_{t,n}]'$ is the vector of residuals.

In the next step, we obtain the conditional volatilities $h_{it}$ from univariate GJR-GARCH(1, 1) process which gives an additional weight to negative returns in order to capture the asymmetric effects. In particular, we estimate the following:

$$h_{it}^2 = \omega + \left(\alpha + \gamma \varepsilon_{t-1} < 0\right)\varepsilon_{it-1}^2 + \beta h_{i,t-1}$$ (2)

where $\gamma$ is the leverage coefficient.

6 By using weekly data, we minimize the adverse effects of belonging to different time zones and having different operating days, yet we do not lose the dynamics of the correlations. Moreover, using indexes disseminated by Dow Jones brings us the advantage that the stocks of the companies included in these indexes satisfy certain liquidity criteria, thus carry more information and can be traded easier compared to stocks of arbitrarily chosen companies following Islamic investment principles.

3.1. Consistent dynamic conditional correlation

The dynamic correlations between the analyzed variables will be obtained by the cDCC model of Aielli (2013). To consider cDCC modeling, we start by reviewing the DCC model of Engle (2002). Assume that for $t=1,\ldots,T$, $E_t-1[\epsilon_t]=0$ and $E_t-1[\epsilon_t\epsilon_t']=H_t$, where $E_t[\cdot]$ is the conditional expectation on $\epsilon_t, \epsilon_{t-1}, \ldots$. The asset conditional covariance matrix $H_t$ can be written as:

$$H_t = D_t^{1/2}R_tD_t^{1/2}$$

where $R_t = [\rho_{ij}]$ is the asset conditional correlation matrix and the diagonal matrix of the asset conditional variances is given by $D_t = diag(h_{t,1}, \ldots, h_{t,n})$. Engle (2002) models the right hand side of Eq. (3) rather than $H_t$ directly and proposes the dynamic correlation structure:

$$R_t = \{Q_t^{-1/2}Q_t^{-1/2}\},$$

$$Q_t = (1-a-b)S + au_{t-1}u_{t-1} + bQ_{t-1}.$$
Table 1
Analyzed Islamic equity sectors and the corresponding Bloomberg tickers.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Ticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic materials</td>
<td>DJIBSC</td>
</tr>
<tr>
<td>Consumer goods</td>
<td>DJINCY</td>
</tr>
<tr>
<td>Consumer services</td>
<td>DJICYC</td>
</tr>
<tr>
<td>Financials</td>
<td>DJIFIN</td>
</tr>
<tr>
<td>Health care</td>
<td>DJIHCR</td>
</tr>
<tr>
<td>Industrials</td>
<td>DJIIDU</td>
</tr>
<tr>
<td>Energy</td>
<td>DJIENE</td>
</tr>
<tr>
<td>Technology</td>
<td>DJITEC</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>DJITLS</td>
</tr>
<tr>
<td>Utilities</td>
<td>DJIUTI</td>
</tr>
</tbody>
</table>

![Graphs of Dow Jones Islamic sector returns from January 1996 to July 2014.](image)

Fig. 2. Dow Jones Islamic sector returns from January 1996 to July 2014.
where \( Q_t \equiv [q_{ij,t}] \), \( u_t = [u_{1,t}, \ldots, u_{n,t}]' \) and \( u_{it} \) is the transformed residuals i.e. \( u_{it} = \epsilon_{it}/h_{it} \), \( S \equiv [s_{ij}] = E[u_{it}u_{it}'] \) is the \( n \times n \) unconditional covariance matrix of \( u_t \), \( Q^*_t = \text{diag}(Q_t) \) and \( a \) and \( b \) are non-negative scalars satisfying \( a + b < 1 \). The final estimation is performed by maximizing the joint log-likelihood of the model given by:

\[
\mathbb{L} = \frac{1}{2} \sum_{t=1}^{T} \left( n \ln(2\pi) + \ln|D_t| + \epsilon_t' R_t^{-1} \epsilon_t \right) - \frac{1}{2} \sum_{t=1}^{T} \left( \ln|R_t| + \epsilon_t' R_t^{-1} \epsilon_t - \epsilon_t' \epsilon_t \right)
\]

(5)

and the resulting model is called DCC.

However, Aielli (2013) shows that the estimation of \( Q \) by this way is inconsistent since \( E[R_t] \neq E[Q_t] \) and he proposes the following consistent model with the correlation driving process:

\[
Q_t = (1-a-b)S + a \left\{ Q_{t-1}^{1/2} u_{t-1} u_t' Q_{t-1}^{1/2} \right\} + bQ_{t-1}
\]

(6)

where \( S \) is the unconditional covariance matrix of \( Q_t^{1/2} u_t \).

3.2. Dynamic equicorrelation

Engle and Kelly (2012) suggest modeling \( \rho_t \) by using the cDCC specification to generate the conditional correlation matrix \( Q_t \) and then taking the mean of its off-diagonal elements as a simplifying procedure to decrease the estimation time. This approach is termed the dynamic equicorrelation (DECO) model, and the scalar equicorrelation is formally defined by:

\[
\rho_t^{\text{DECO}} = \frac{1}{n(n-1)} \left( J_n R_t^{\text{cDCC}} J_n - n \right) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}
\]

(7)

where \( q_{ij,t} \) is the \((i,j)^{th}\) element of the matrix \( Q_t \) from the cDCC model. This scalar equicorrelation is then used to create the conditional correlation matrix:

\[
R_t = (1-\rho_t)I_n + \rho_t J_n
\]

(8)

where \( J_n \) is the \( n \times n \) matrix of ones and \( I_n \) is the \( n \)-dimensional identity matrix. This assumption of equicorrelation leads to a much simpler likelihood equation when \( \rho_t \) is given by Eq. (7):

\[
\mathbb{L} = -\frac{1}{2} \sum_{t=1}^{T} \left( \ln((1-\rho_t)^{n-1}(1+(n-1)\rho_t)) + \frac{1}{1-\rho_t} \left( \sum_{i=1}^{n} \epsilon_{i,t}^2 - \frac{\rho_t}{1+(n-1)\rho_t} \left( \sum_{i=1}^{n} \epsilon_{i,t}^2 \right)^2 \right) \right).
\]

(9)

In the new structure, one avoids the inversion of the matrix \( R_t \), so DECO is less burdensome and computationally quicker to estimate. Besides, this process helps us to represent the co-movement degree of a group of assets with a single time-varying correlation coefficient.

3.3. Advantages of time-varying conditional correlation approach

Instead of the common approach of rolling window Pearson correlations, we use time-varying conditional correlation models in this study. The advantages are two-fold:

First, in the rolling window approach, the results are heavily autocorrelated due to the overlapping windows and the choice of the window length and the rolling step can be controversial. The problem of overlapping can be overcome by using non-overlapping windows. However, in this case the problem of window size is still a big problem and moreover another problem also arises. For example, in

\(^{8}\) In this study, we also performed our analysis with the DCC approaches of both Engle (2002) and Tse and Tsui (2002) by implementing various conditional volatility modelings such as GARCH, APARCH and FIGARCH; however we observed no significant difference in the outcome. We do not provide the quantitative results in the manuscript, however they can be obtained upon request.
our analysis, we use approximately 20 years of weekly data. In total, we have 810 weeks of price series for each sector, thus 809 returns i.e. 809 data points. Now, to have a statistically significant correlation between the time series, how many data points do we need? For example, if we take 52 points as the window size (which corresponds to a year in our analysis) then the number of non-overlapping windows is going to be 15, which is very small and really meaningless in our case. To have more time varying correlation data, we need to shorten the window size: for example, for 25 points (6 months) the number of non-overlapping windows is 32, which again is a very small number for our analysis. And, for shorter window sizes, Pearson correlations will start to be meaningless. In the cDCC case, every week (from the beginning to the end) is associated with a correlation level without consuming any initial data.

Second, there is a heteroskedasticity problem when estimating Pearson correlations, caused by volatility increases during the crisis. For example, if a crisis hits country A with increasing volatility in its stock market, it will be transmitted to Country B with a rise in volatility and, in turn, the correlation of stock returns in both Country A and B. This is overcome by the cDCC model since it estimates correlation coefficients of the standardized returns and thus accounts for heteroskedasticity directly.

Due to its advantages, the DCC methodology has become the main tool in the last decade in analyzing the contemporaneous relationship between the financial time series. For the recent applications of cDCC approach, see Boudt et al. (2013), Aslanidis and Casas (2013), Teulon et al. (2014), Sobaci et al. (2014), Sensoy et al. (2014). For the applications of DECO model, see Sensoy (2013), Christoffersen et al. (2014), Aboura and Chevallier (2014).

4. Results and discussion

Table 2 presents the statistical properties of the returns as well as the unit root test results. We can see that consumer services sector has the highest weekly average return over the study period, followed by health care...
and energy sectors, whereas utilities and financials present the lowest mean returns. Moreover, the changes in the financial sector prices exhibit almost zero returns on a weekly basis.

The unconditional volatilities of the sector indexes, measured by standard deviations, seem not so different from each other in general, with the exception of consumer goods which has a relatively significant lower unconditional volatility among all others. Return distributions are skewed to the left for all the indexes. Also, all sector returns exhibit excess kurtosis (fat tails). Skewness and kurtosis coefficients indicate that return series are far from normally distributed. This departure from normality is formally confirmed by the Jarque–Bera test statistics that rejects normality at the 1% level for all series.9

Table 2 also presents the results of the conventional stationarity test for our return series (unit root tests contain a constant). Augmented Dickey–Fuller (ADF) test rejects the null hypothesis of unit root for all the return series at the 1% significance level. Similarly, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test can not reject the stationarity of the returns at the 1% significance level. All the return series are therefore stationary.

The estimation results for the mean equation and the GJR-GARCH model are presented in Table 3. Table 3 shows that the tail parameter β is statistically significant for each sector index, which confirms the existence of the leptokurtic behavior of return series. Except the telecommunications and utilities sectors, strong evidence of volatility asymmetry is observed in each series as the parameter (γ) is statistically significant at 1% level, which validates the appropriateness of the GJR-GARCH to model the volatility of the return series.

Fig. 3 displays the dynamic equicorrelation level for the Islamic equity market sectors and Table 4 presents the corresponding parameter estimations. We further filter this correlation by Hodrick and Prescott (1997) process to eliminate the noise in the data and to clearly visualize the trend component.10 Fig. 3 also exhibits the mean breaks (according to penalized contrast functions) in the dynamic equicorrelation level, and the corresponding break dates/directions are presented in Table 5.11

The trend component presents striking patterns: the first noteworthy change takes place around August 1999 where dynamic equicorrelation drops significantly from its stable levels around 0.5 to the levels around 0.25.12 The low equicorrelation level continues for twenty months until it jumps back to its previous values in March 2001. A careful reader can easily associate this period with the “collapse of the dot-com bubble” which took place during 1999–2001. The dot-com bubble was a historic speculative bubble covering roughly 1997–1999 in which stock markets of industrialized nations saw their equity prices rose rapidly due to growth in the technology sector and related fields. Due to its nature, it was a self-perpetuating rise in the share prices of technology stocks, however the prices of many non-technology stocks increased in tandem and were also pushed up to unrealistic valuations relative to fundamentals (Anderson et al., 2010). Although an in-depth analysis will be provided shortly, the behavior of the equicorrelation trend initially suggests that during 1999–2001, the equities in the heavily financialized sectors were affected by the bubble and hence their prices and the prices of the equities in the sectors where financialization stays limited have diverged from each other (for example, see Fig. 1:

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9 cDCC model estimation is still valid under non-normality of weekly changes in our time series i.e. the assumption of normality is not required for consistency and asymptotic normality of the estimated parameters. When the return series have non-Gaussian distributions, the cDCC estimator can be interpreted as a quasi-maximum likelihood estimator. For further details see Engle and Sheppard (2001).

10 This filtering uses ideas related to the decomposition of time series: Let yt for t = 1, 2, ..., T denote the logarithms of a time series variable. The series yt is made up of a trend component, denoted by τ and noise c such that yt = τt + ct. Given an adequately chosen positive λ, there is a trend component that solves \(\min_{\tau} \sum_{t=1}^{T} \left(y_{t} - \tau_{t}\right)^{2} + \lambda \sum_{t=2}^{T} \left(\tau_{t+1} - \tau_{t} - (\tau_{t} - \tau_{t-1})\right)^{2}\). The first term of the equation is the sum of the squared deviations \(d_{t} = y_{t} - \tau_{t}\) which penalizes the noise. The second term is a multiple λ of the sum of the squares of the trend component’s second differences. This second term penalizes variations in the growth rate of the trend component. The larger the value of λ, the higher is the penalty (Hodrick and Prescott, 1997). A common choice for weekly data is \(\lambda = 100 \times 52^{2} = 270400\), and we use this value in our analysis.

11 We use penalized contrast functions to endogenously determine the breakdowns in dynamic equicorrelation. Compared to its alternatives, one of the main advantages of using a penalized contrast methodology is that the variables are not necessarily normally distributed or independent, moreover it is less sensitive to outliers. Since it is not the main methodology of this paper, we present it in the Appendix A.

12 Different than sample Pearson correlations, DECO level at a chosen point does not stand for a long term correlation, but is more like a representative of a comovement level on a given point. On the other hand, interpreting whether a DECO level is high or low is no different than interpreting a regular correlation level. But more importantly, a DECO level being high or low should be compared to its past or future values as the main purpose of the methodology is to obtain a time varying comovement degree.
While sectors such as financials, industrials, technology and telecommunications were heavily affected by the dot-com bubble, the effect seems relatively limited for the basic materials, consumer goods, health care and energy sectors.

The second interesting pattern of the trend component can be observed during 2002–2008. Accordingly, we observe a gradually increasing co-movement degree between Islamic equity sectors lasting for six to seven years. Such a pattern is indeed expected to be observed during this era: with the beginning of the new millennium, the financial system around the world started to become more complex than ever. With the introduction of several mutual funds/ETFs and with the help of the financial innovations, the investment strategies have changed completely. Moreover, the integration of emerging countries to the global capital

### Table 3
Parameter estimates for the mean and variance equations.

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\mu$</th>
<th>$\omega \times 10^4$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic materials</td>
<td>0.0012</td>
<td>0.2471**</td>
<td>0.0622**</td>
<td>0.8538***</td>
<td>0.1193***</td>
</tr>
<tr>
<td></td>
<td>(0.120)</td>
<td>(0.028)</td>
<td>(0.034)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Consumer goods</td>
<td>0.0015***</td>
<td>0.2516**</td>
<td>-0.0055</td>
<td>0.7967***</td>
<td>0.2735***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.019)</td>
<td>(0.825)</td>
<td>(0.000)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Consumer services</td>
<td>0.0018***</td>
<td>0.2950**</td>
<td>0.0249</td>
<td>0.8112***</td>
<td>0.2351***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.014)</td>
<td>(0.208)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Financials</td>
<td>0.0011</td>
<td>0.1663*</td>
<td>0.0552**</td>
<td>0.8685***</td>
<td>0.1267**</td>
</tr>
<tr>
<td></td>
<td>(0.161)</td>
<td>(0.054)</td>
<td>(0.034)</td>
<td>(0.000)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>Health care</td>
<td>0.0010***</td>
<td>0.6701***</td>
<td>0.0319</td>
<td>0.6369***</td>
<td>0.3859***</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.005)</td>
<td>(0.478)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Industrials</td>
<td>0.0017**</td>
<td>0.2540***</td>
<td>0.0539**</td>
<td>0.8195***</td>
<td>0.1759***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.006)</td>
<td>(0.022)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Energy</td>
<td>0.0014</td>
<td>0.4587**</td>
<td>-0.0018</td>
<td>0.8601***</td>
<td>0.1654***</td>
</tr>
<tr>
<td></td>
<td>(0.107)</td>
<td>(0.028)</td>
<td>(0.946)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Technology</td>
<td>0.0016*</td>
<td>0.1667**</td>
<td>0.0427***</td>
<td>0.8854***</td>
<td>0.1144***</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.033)</td>
<td>(0.902)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>0.0010</td>
<td>0.1994**</td>
<td>0.1013***</td>
<td>0.8391***</td>
<td>0.0670</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.037)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.0014**</td>
<td>0.3542***</td>
<td>0.1177***</td>
<td>0.7555***</td>
<td>0.1246</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.009)</td>
<td>(0.003)</td>
<td>(0.000)</td>
<td>(0.103)</td>
</tr>
</tbody>
</table>

1. Mean equation: $\tau_t = \mu + \epsilon_t$.
2. Variance equation: $h_t^2 = \omega + (\alpha + \gamma \epsilon_t - 1 < 0)\epsilon_t^2 - 1 + \beta h_{t-1}^2$.
3. The values in the parenthesis are $p$-values obtained from robust standard errors.

While sectors such as financials, industrials, technology and telecommunications were heavily affected by the dot-com bubble, the effect seems relatively limited for the basic materials, consumer goods, health care and energy sectors.

The second interesting pattern of the trend component can be observed during 2002–2008. Accordingly, we observe a gradually increasing co-movement degree between Islamic equity sectors lasting for six to seven years. Such a pattern is indeed expected to be observed during this era: with the beginning of the new millennium, the financial system around the world started to become more complex than ever. With the introduction of several mutual funds/ETFs and with the help of the financial innovations, the investment strategies have changed completely. Moreover, the integration of emerging countries to the global capital

![Islamic Market Sectors](image-url)

**Fig. 3.** Dynamic equicorrelation of Islamic equity market sectors. Vertical lines (arrows) denote the mean breaks (break direction) in the dynamic equicorrelation level.
markets and the removal of barriers to international capital flows created a level of demand for financial instruments that has never been experienced before. Such a demand brought out the financialization period worldwide where firm fundamentals and real supply/demand factors were no more the main driving force of the related equity prices (for example, see Fig. 1: Without an exception, each sector’s price sits on an increasing path during 2002–2007). Instead, the prices started to be affected mostly by the investment decisions given in order to construct portfolio management strategies.\(^{13}\) With the low risk–high return aim in mind, the investment strategies looked for low correlated equities, however as Authors (2010) states, the more investors buy assets on the assumption that they are not correlated, the more they tend to become correlated which may be thought as the “paradox of diversification”, eventually leading to an integration in several Islamic equity sectors and an immediate response of sector prices to information captured in other financial asset prices.

The third main pattern actually arises inevitably after the second one: in October 2008, following the collapse of the Lehman Brothers which may be considered as the peak of the most dangerous crisis since the Great Depression of the 1930s, the group co-movement degree of Islamic equity sectors jumps to a new level around 0.75 and fluctuates around this value since then. Although many causes for the financial crisis have been suggested (such as high risk seeking behavior by investors, complex financial products, undisclosed conflicts of interest, the failure of regulators, the credit rating agencies), from a result oriented perspective, the worldwide financial system ended up with a systemic meltdown causing all sector prices to drop significantly and naturally, leading to a highly increased group co-movement level. Since then, the dynamic equicorrelation trend preserves its high level values suggesting an ongoing synchronization between Islamic equity sectors possibly due to the “never really ending” turbulent times in financial markets.

The interesting patterns displayed by dynamic equicorrelation and its trend component motivate us further to analyze the interaction structure between Islamic equity sectors. Such a detailed analysis requires the investigation of every time-varying pairwise relationship. In order to do that, we estimate the dynamic conditional correlation between each pair of individual sectors by cDCC methodology of Aielli (2013). Dynamic correlations between each pair of sectors are illustrated in Fig. 4, and the estimated parameters are given in Table 4.

\[^{13}\] For more on the concept of financialization, see Tang and Xiong (2012) and Buyuk Sahin and Robe (2014).

### Table 4
Driving parameters of DECO and cDCC processes.

<table>
<thead>
<tr>
<th></th>
<th>cDCC</th>
<th>DECO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Islamic equity sectors</td>
<td>0.0225***</td>
<td>0.0937**</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td>0.9658***</td>
<td>0.9012***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

1. cDCC process is driven by \( Q_t = (1 – a - b)S + a(Q_{t-1}^{1/2}u_{t-1} – u_{t-1} – Q_{t-1}^{1/2}) + bQ_{t-1} \).
2. The DECO is obtained by \( \rho_{DECO}^t = \frac{1}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{q_{ij}}{\sqrt{q_{ii}q_{jj}}} \) where \( q_{ij} \) is the \((i,j)\)th element of the matrix \( Q_t \) from the cDCC model. This scalar equicorrelation is then used to create the conditional equicorrelation matrix \( R_t = (1 - \rho_t)I_n + \rho_t J_n \).
3. The values in the parentheses are the \( p \)-values obtained from robust standard errors.

### Table 5
Mean break dates/directions of the dynamic equicorrelation between Islamic equity market sectors.

<table>
<thead>
<tr>
<th>Date</th>
<th>( \rho_{DECO} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>04/08/1999</td>
<td>Down</td>
</tr>
<tr>
<td>14/03/2001</td>
<td>Up</td>
</tr>
<tr>
<td>08/10/2008</td>
<td>Up</td>
</tr>
</tbody>
</table>
Fig. 4 provides us the complete interaction structure that we require for our analysis, and moreover strengthens our previous deductions on these interactions. Considering the first noteworthy change in the trend component of the equicorrelation (where we observe a significant drop during 1999–2001), we see that for the pairs constructed by picking one sector from basic materials, consumer goods and health care; and another sector from financials, industrials, energy, technology, telecommunications and utilities, there is a significant downwards shift in bilateral correlations during 1999–2001. This is in parallel to our assumptions that the equities in the heavily financialized sectors (and the ones mostly related to technology sector) were severely affected by the dot-com bubble, thus their prices and the prices of the equities in the less financialized sectors in that period have diverged from each other.

Regarding the second main pattern (where the trend component of the equicorrelation gradually increases during 2002–2008), we observe an increasing degree (to varying extents) of co-movement between almost every pair during 2002–2008 in our analysis. This situation favors our previous argument on the worldwide financialization period where capital flows and investment strategies shape the related equity prices rather than firm fundamentals and real product supply/demand factors, which eventually creates an overall integration of Islamic equity sectors.

Finally, regarding the third main pattern (where trend component shifts significantly to an upper level and fluctuates around that level since then), there are different structures in the pairwise interactions: the striking part about the third pattern is the upward jump in the dynamic equicorrelation level, however, according to our analysis this phenomena arises due to limited number of pairs. In particular, for most of the pairs where financials, energy or utilities is included as one of the pair components, we observe a noticeable upward break in the bilateral correlations around the year 2008, however this is not a common conclusion for the remaining pairs. Moreover, and interestingly, we do not observe a significant drop in the correlations between some specific sectors in contrast to the case of 1999–2001, which validates our claim on the price driving dynamics in the last decade even more.

Other than the main patterns, an interesting interaction structure is observed for a very limited number of pairs i.e. consumer services–industrials, consumer services–telecommunications, financials–industrials, financials–technology, industrials–technology, industrials–telecommunications and technology–telecommunications. For these pairs, bilateral correlations display almost no significant change since the late 1990s. The situation suggests that these sectors have already been integrated up to some level even in the 1990s, possibly due to their close relevancy with each other and their early financialization process in the global financial system.

Overall, although we consider several different types of Islamic equity sectors, the correlation dynamics show us that all together they tend to converge to a single asset class in the last decade.

An interesting field to be explored at this point would be the analysis of the same cross-sectoral relationships in the conventional equity markets and make a detailed comparison. Although this is not the aim of this study, we provide a general view to this comparison for the interested reader by displaying the dynamic equicorrelation and its trend component for the conventional equity sectors (exactly the same sectors that have been analyzed previously). Similar to the previous case, we use the data disseminated by the Dow Jones and further, to eliminate any biases, we employ exactly the same methodological setup as we did before. However, the weekly conventional sector indexes are available from the beginning of the year 1999. Therefore, the initial date of the sample conventional data is not an exact match for its Islamic counterpart. Fig. 5 illustrates the dynamic equicorrelations and their trend components for both Islamic and conventional equity market sectors, and presents a noteworthy difference in the main correlation trend structures. Accordingly, the sectoral integration in conventional equity markets seems to have taken place much earlier than the Islamic equity markets. Regarding the conventional part, the global financial crisis of the 2008 has made less of an impact on the sector integration as the level of integration (trend component) was already considerably high prior to the catastrophic events that took place in 2008. Although, interestingly, the level of difference in cross-sectoral integration in Islamic and conventional equity markets clearly has diminished during the period following the peak of the crisis. Such an observation further supports our argument on the impact of financialization on Islamic equity sector interaction and integration.

---

14 Meaning that sector prices behave similarly in the market and consistently correlate highly with one another.
5. Conclusion

Many organizations have introduced several regional and global equity indexes for Shari’ah compliant firms lately, and the recent increase in the number of these indexes is in line with the growing demand for Shari’ah compliant investment instruments. Although there is a growing literature on the performance

![Fig. 4. Consistent dynamic conditional correlations between each pair of Islamic equity market sectors.](image)
comparison between Islamic and conventional financial systems and the interactions in-between, the number of studies analyzing the with-in interactions of Islamic equities stays very limited. In this study, to fill this gap, we analyzed the correspondence between the different sectors belonging to Dow Jones Islamic equity indexes using several state of the art techniques.
Even though the business activity must be compliant with Shari’ah in order to be included in Islamic equity indexes in theory, firms included in the Dow Jones Islamic indexes do not strictly fulfill this requirement as Islamic scholars have made some concessions on the permissible degree of financial leverage and interest earnings for constituent firms in these indexes. Moreover, they are not strictly immune to widespread impacts.
of interest rate changes because of economic interdependence among economic agents with different ethical values.\textsuperscript{15} This situation brings out the inevitable necessity for portfolio diversification for the investors who want to reconcile faith with finance. Regarding equity investment strategies in this context, one of the first things that comes to mind is to construct a portfolio that is well diversified among different sectors. As Shamsuddin (2014) suggests, such an aggregate portfolio is the key to become immune to common interest rate risk for investors who want to comply with their religious obligations. However, our results show that such a diversification among different Islamic equity sectors may not be beneficial anymore lately. We have shown that after the global financialization, the price driving force of the fundamentals seemed to disappear as the fast profit making approach through financial markets started to dominate over the traditional indicators to price equities, leading to a high level of sensitivity to the information captured in other asset prices and, inevitably, highly integrated Islamic equity sectors just as in the case of the conventional part. In other words, investors’ moving away from passive strategies and opting for active ones resulted with the proliferation of global Islamic equity indexes and the dramatic increase of investment to these Islamic equities through index tracking funds. As mutual funds and ETFs that track global Islamic equity indexes have become popular, Islamic equities that once traded largely independently of each other now get lumped together and bought–sold as a group i.e. all the components are traded indiscriminately, regardless of their true value and/or firm fundamentals, yielding to a severe increase in correlations even between the different Islamic equity sectors.

\textsuperscript{15} For example, see Ajmi et al. (2014) for evidence of non-linear causal relationship between Islamic and conventional markets. Moreover, a recent study by Hammoudeh et al. (2014) reveals significant dependence between Dow Jones global Islamic Market equity index and the three major global conventional equity indexes (Asia, Europe and US).
Moreover, this effect is emphasized further through financial contagion channels in the recent global financial crisis. Therefore, this paper presents three-fold suggestions in terms of policy implications:

First, due to the excess global financialization, Islamic equity indexes are also prone to global shocks hitting the world financial system through massive capital movements and investor sentiment. Keeping this fact in mind, investors, portfolio managers and policy makers should be cautious in interpreting and forecasting the interaction structure between Islamic equity sectors as they are no longer completely determined with the related firm or sector fundamentals.

Second, due to highly increased level of correlations between Islamic equity sectors (although still lower than in the case of the conventional system), the benefit of sectoral diversification has diminished significantly in the last decade hence the investors looking for financial strategies within Shari’ah compliance should definitely include financial instruments from other asset classes in their portfolios.

Third, recent studies favor the weakening of the decoupling hypothesis of Islamic equity finance from its conventional counterpart as the Shari’ah screening cannot fully eliminate the economic interdependence among agents with different ethical values for several reasons, and this weakening is partially validated in our study, at least at the cross-sectoral integration level. If Islamic equity markets still want to provide a cushion against financial shocks affecting the conventional markets, authorities should consider establishing a worldwide integrated Islamic financial market system isolated from conventional dynamics as much as possible.

We believe that these findings will be of great importance to investors, portfolio managers, policy makers and Islamic finance authorities.

Acknowledgments

We are grateful to the editors S. G. Rhee, Mansor H. Ibrahim and an anonymous referee for appropriate and constructive comments/suggestions that significantly improved this paper.

Appendix A. Detection of mean breaks in the DECO

To detect the dates of mean breaks in the dynamic equicorrelation level, we use penalized contrasts. To prevent misunderstandings, the reader is asked to consider the mathematical notations in Appendix A independent from the other parts of this manuscript.

Consider a sequence of random variables \( Y_1, \ldots, Y_n \) that take values in \( \mathbb{R}^p \). Assume that \( \theta \in \Theta \) is a parameter denoting the characteristics of the \( Y_i \)'s that changes abruptly and remains constant between two changes. Change occurs at some instants \( \tau_1 < \tau_2 < \ldots < \tau_K - 1 \). Here \( K^* - 1 \) is the number of change points hence we have \( K^* \) number of segments where \( * \) is used to denote the true value. Now, let \( K \) be some integer and let \( \tau = (\tau_1, \tau_2, \ldots, \tau_K - 1) \) be a sequence of integers satisfying \( 0 < \tau_1 < \tau_2 < \ldots < \tau_K - 1 < n \). For any \( 1 \leq k \leq K \), let \( U(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}) \) be a contrast function useful for estimating the unknown true value of the parameter in the segment \( k \); i.e. the minimum contrast estimate \( \hat{\theta}(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}) \), computed on segment \( k \) of \( \tau \), is defined as a solution of the following minimization problem:

\[
U(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}; \hat{\theta}(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k})) \leq U(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}; \theta), \quad \forall \theta \in \Theta. \tag{A.1}
\]

For any \( 1 \leq k \leq K \), let \( G \) be

\[
G(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}) = U(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}; \hat{\theta}(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k})) \tag{A.2}
\]

then define the contrast function \( f(\tau, y) \) as

\[
f(\tau, y) = \frac{1}{n} \sum_{k=1}^{K} G(Y_{\tau_{k-1} + 1}^\tau, \ldots, Y_{\tau_k}) \tag{A.3}
\]
where \( \tau_0 = 0 \) and \( rk = n \). When true number \( K^* \) segments is known, for any \( 1 \leq k \leq K^* \), the sequence \( \hat{\tau}_n \) of change point instants that minimizes this kind of contrast has the property that

\[
\Pr \left( \left| \hat{\tau}_{n,k} - \tau_k^* \right| > \delta \right) \to 0, \text{ when } \delta \to \infty \text{ and } n \to \infty. \tag{A.5}
\]

In particular, this result holds for weak or strong dependent processes. We consider the model \( Y_i = \mu_i + \epsilon_i \), \( 1 \leq i \leq n \), where \( (\epsilon_i) \) is a sequence zero-mean random variables with unit variance. In the case of detecting changes in the mean, we assume that \( (\mu_i) \) is a piecewise constant sequence and \( (\epsilon_i) \) is a constant sequence. Now, even if \( (\epsilon_i) \) is not normally distributed, a Gaussian log-likelihood can be used to define the contrast function. Let

\[
U(Y_{\tau_{k-1}+1}, \ldots, Y_{\tau_k}; \mu) = \sum_{i=\tau_{k-1}+1}^{\tau_k} (Y_i - \mu)^2 \tag{A.5}
\]

then

\[
G(Y_{\tau_{k-1}+1}, \ldots, Y_{\tau_k}) = \sum_{i=\tau_{k-1}+1}^{\tau_k} (Y_i - \bar{Y}_{\tau_{k-1}+1, \tau_k})^2 \tag{A.6}
\]

where \( \bar{Y}_{\tau_{k-1}+1, \tau_k} \) is the empirical mean of \( (Y_{\tau_{k-1}+1}, \ldots, Y_{\tau_k}) \).

When the number of shift points is unknown, it is estimated by minimizing a penalized version of \( J(\tau, \mathbf{y}) \). For any sequence of change point instants \( \tau \), let \( pen(\tau) \) be a function of \( \tau \) that increases with the number \( K(\tau) \) of segments of \( \tau \). Then, let \( \hat{\tau}_n \) be the sequence of change point instants that minimizes:

\[
F(\tau) = J(\tau, \mathbf{y}) + \phi pen(\tau) \tag{A.7}
\]

where \( \phi \) is a function of \( n \) that goes to zero at an appropriate rate as \( n \) goes to infinity. The estimated number of segments \( K(\hat{\tau}_n) \) converges in probability to \( K^* \). The proper \( pen(\tau) \) and the penalization parameter \( \phi \) are chosen according to Lavielle (2005). For further details, refer to Lavielle (2005).

References


